Rice University
PHYS 112

Induction

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Faraday's Law of Induction is analyzed with a set of two coils with one driven by a function generator. Additionally, a transformer is driven and analyzed.

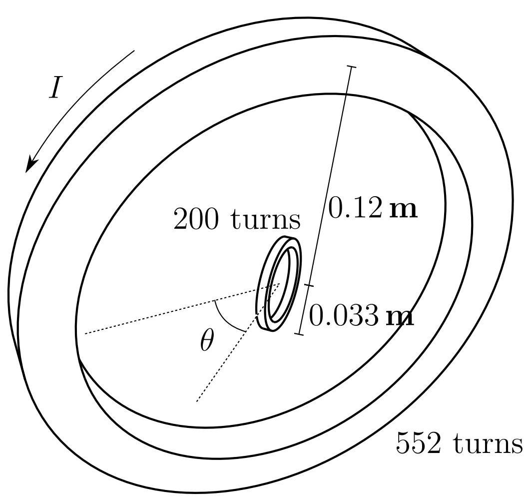
1 Faraday's Law

Faraday's Law observes that a changing magnetic field induces a current in nearby conductors. The induced electromotive force through a loop of wire is the negative rate of change of the magnetic flux through the loop's opening.

1.1 Apparatus

The following apparatus was analyzed.

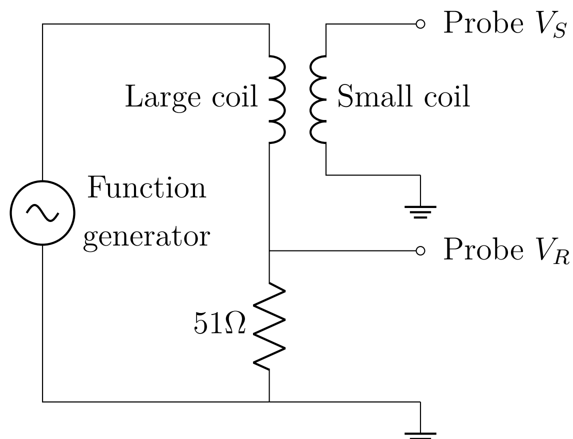
Figure 1: Apparatus for Faraday's Law



A small coil of wire is placed inside a larger coil, free to be rotated to a fixed angle θ against the larger coil. The number of turns for each coil are indicated, as well as their radii.

A current passed through the large coil creates a magnetic field which is close to uniform in the neighborhood of the small coil.

Figure 2: Circuit schematic for apparatus



The larger coil was driven by a function generator. A 51Ω resistor was placed so its voltage $V_R(t)$ was proportional to the current in the large coil. Voltage $V_S(t)$ over the small coil was directly observed.

1.2 Expected Behavior

The resistor of resistance R ensures that the current I_p in the driving coil can be measured by observing the voltage V_R over the resistor.

$$I_p = \frac{V_R}{R}$$

A current through the large coil produces a magnetic field which can be approximated as uniform at the center of the coil. For current I_p in the large coil with N_p turns and average radius a_p, the magnetic field at the center has magnitude B such that

$$B = \frac{\mu_0 N_p I_p}{2a_p}$$

The flux through the smaller coil of N_S turns and radius a_S is

$$\Phi_B = \frac{1}{2}\left(2\pi\right) a_S^2 B N_S \cos\theta$$

By Faraday's Law, the induced voltage V_S in the small coil is such that

$$V_S = -\frac{d\Phi_B}{dt}$$

Putting everything together, if the large coil is driven with a sine wave of angular frequency ω such that for *peak* voltage V_R over the resistor,

$$I_p(t) = \frac{V_R}{R}\sin(\omega t)$$

then following induced voltage in the small coil is expected.

$$V_S(t) = \frac{\left(2\pi\right)\mu_0 N_p N_S a_S^2 \omega V_R}{4a_p R}\cos(\omega t)\cos\theta \tag{1}$$

1.3 Observations

By Equation 1, the induced voltage depends on several parameters of the apparatus.

1.3.1 Current dependence

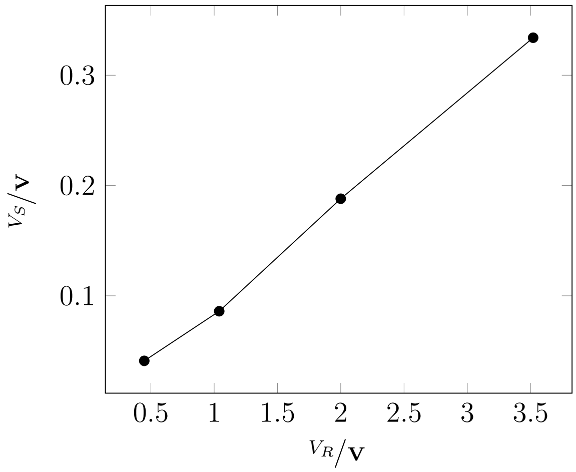
Equation 1 states that the induced voltage is proportional to the peak voltage over the resistor, which is itself proportional to the peak current passed through the driving coil. By equation 1, the proportionality constant between peak voltages V_S and V_R can be quantified as

$$\frac{\left(2\pi\right)\mu_0 N_p N_S a_S^2 \omega}{4a_p R}\cos\theta$$

The large coil was driven with an angular frequency of 2563.5 **Hz** at varying currents. The coil angle θ was held flush at 0. Thus, the proportionality constant is predicted to be abut 0.0993.

The following results indicate the peak voltage readings of V_R and V_S for different current amplitudes.

Figure 3: Peak induced V_S as a function of peak resistor voltage V_R

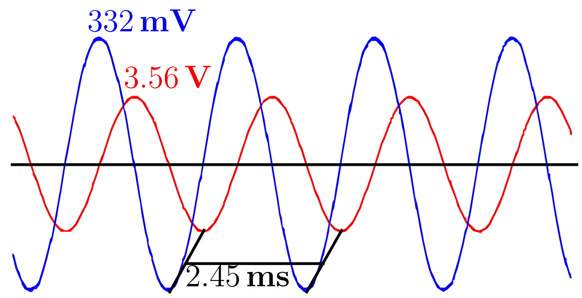


It is seen that V_S is indeed quite proportional to V_R ($R^2 = 0.998$) with a slope of 0.097 ± 0.015. This agrees with the expected value of 0.0993.

As the induced voltage relies on the time derivative of the driving voltage, the induced voltage is expected to lag behind the driving voltage by a quarter turn.

Here are the actual oscilloscope readings with $V_S(t)$ indicated in blue and $V_R(t)$ indicated in red (relative voltage scale adjusted for readability)

Figure 4: Driving voltage vs induced voltage over time



With both waves at the same frequency, the quarter turn lad is indeed observed, as indicated by each peak/trough of one graph corresponding with a zero-point of the other.

1.3.2 Frequency Dependence

Equation 1 states that the induced voltage is also proportional to the frequency of the driving voltage. By the equation, the proportionality constant between peak voltage V_S and angular frequency ω can be quantified as

$$\frac{(2\pi)\,\mu_0 N_p N_S a_S^2 V_R}{4 a_p R} \cos\theta$$

The large coil was driven with a range of different angular frequencies ranging from about 600 **Hz** to about 13 **kHz**. Below are the measurements of induced voltage V_S, as long as the calculated values of V_S from ω and V_R, denoted $\hat{V_S}$.

Table 1: Induced voltage for different frequencies ω and driving voltages V_R

$\omega/\mathbf{Hz}$	$V_R/\mathbf{V}$	$V_S/\mathbf{V}$	$\hat{V_S}/\mathbf{V}$
628	8.24	0.198	0.201
1275	5.92	0.286	0.293
3160	2.96	0.344	0.363
6315	1.62	0.358	0.397
12623	0.760	0.362	0.372

The prediction gives an average percent error of about 4.6%. While not perfect, it's a reasonable prediction given the precision limitations of the apparatus versus the idealized approximations made to model it.

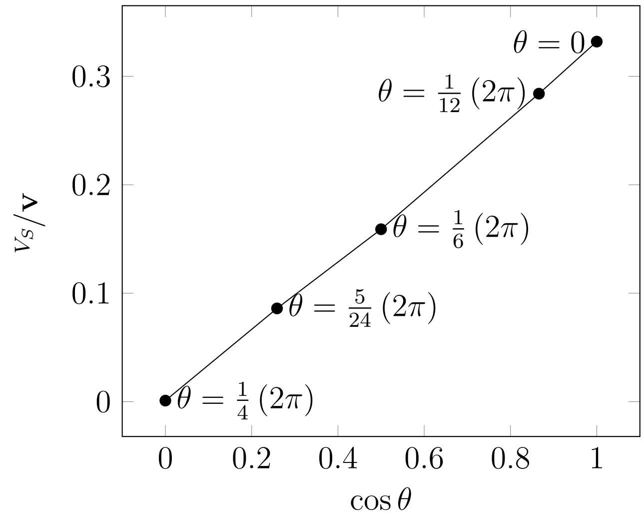
1.3.3 Angle Dependence

Equation 1 states that the induced voltage is also proportional to the cosine of the angle between the coils. By the equation, the proportionality constant between peak voltage V_S and cosine quantity $\cos\theta$ can be quantified as

$$\frac{(2\pi)\,\mu_0 N_p N_S a_S^2 \omega V_R}{4 a_p R}$$

The large coil was driven at an angular frequency of 2601 **Hz** and some arbitrary set voltage resulting in the resistor voltage peaking at V_R. The small coil was rotated some angle θ against the large coil for several runs.

Figure 5: Induced voltage for different angles θ between the coils

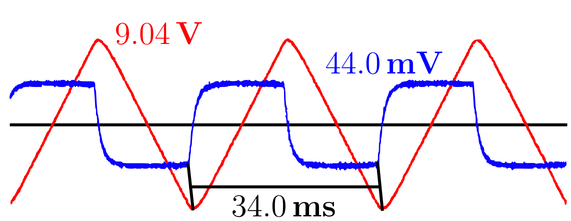


The plot demonstrates the proportionality between the amplitude of V_S and $\cos\theta$ as expected.

1.3.4 Triangular Wave Drive

The large coil was then driven by a triangular wave in place of a sine.

Figure 6: Triangular wave driving current



The induced voltage traces out what approximates a square wave with changes occurring at the peaks and troughs of the driving triangular wave.

This makes sense. The induced voltage relies on the time derivative of the magnetic flux, making the induced voltage proportional to the time derivative of the magnetic field, and thus the time derivative of the driving current / voltage. The time derivative of a triangular wave is a square wave, as each linear section has constant slope.

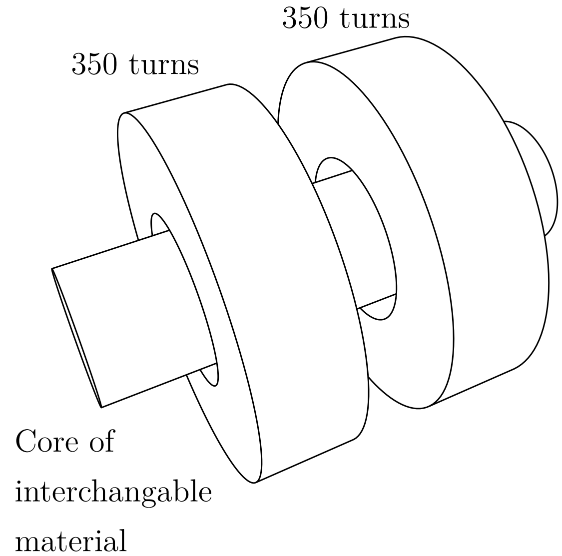
2 Transformers

A transformer is a set of two coils placed in close proximity, used to change voltage and impedance levels arbitrarily.

2.1 Construction

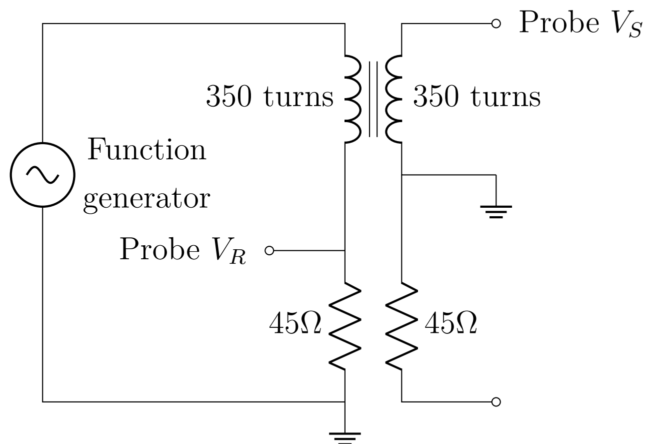
A transformer was constructed by placing two coils of wire side by side. Space was left in the center of the coils to allow a core to be placed inside of any material to be tested.

Figure 7: The transformer constructed



The transformer was linked to a function generator and resistors to measure the voltage over each coil with an oscilloscope.

Figure 8: Circuit schematic for transformer



2.2 Expected Behavior

When one coil is driven by current I_p, the induced voltage V_S in the other coil V_S is proportional to the time derivative of the driving current. The mutual inductance M between the coils is defined such that

$$V_S(t) = -M\frac{dI_p(t)}{dt}$$

With V_R known and R set to 45Ω, the driving current I_p is obtained:

$$I_p = \frac{V_R}{R}$$

The amplitude of the time derivative of the sinusoidal driving current is related to the amplitude of the current itself by a factor of ω due to the sinusoidal nature. Thus, speaking in amplitudes,

$$\frac{dI_p}{dt} = \frac{\omega}{R}V_R$$

This allows the mutual inductance M to be calculated by relating the time derivative of the current to the induced coil voltage.

Finally, an ideal transformer is known to have the following relation:

$$\frac{V_S(t)}{V_p(t)} = \frac{N_S}{N_p}$$

The transformer constructed is not ideal, but if it were, V_S would equal V_p due to the equal number of turns through each coil.

2.3 Observations

Measurements of V_S and V_R were taken over a range of frequencies with different materials of core including air, solid iron, and a bundle of iron wire. The table is populated with the calculated amplitudes of $\frac{dI_p}{dt}$ as well as the mutual inductance M calculated from that.

Table 2: Transformer readings

Core	$\omega/\mathbf{Hz}$	$V_R/\mathbf{V}$	$V_S/\mathbf{V}$	$\frac{dI_p}{dt}\frac{\mathbf{s}}{\mathbf{A}}$	$M/\mathbf{H}$
Air	628	5.72	0.0272	79.87	0.000 341
Air	6 280	5.64	0.272	787.5	0.000 345
Air	62 800	4.24	2.04	5920	0.000 346
Air	628 000	0.628	3.24	8768	0.000 370
Solid Iron Bar	628	5.64	0.400	78.75	0.005 08
Solid Iron Bar	6 280	4.88	2.24	681.4	0.003 29
Solid Iron Bar	62 800	2.24	4.24	3128	0.001 36
Solid Iron Bar	628 000	0.516	3.36	7205	0.000 466
Bundle Of Wires	628	5.04	0.440	70.37	0.006 25
Bundle Of Wires	6 280	5.00	3.75	698.1	0.005 37
Bundle Of Wires	62 800	1.28	7.68	1787	0.004 30
Bundle Of Wires	628 000	0.210	7.60	2932	0.002 59

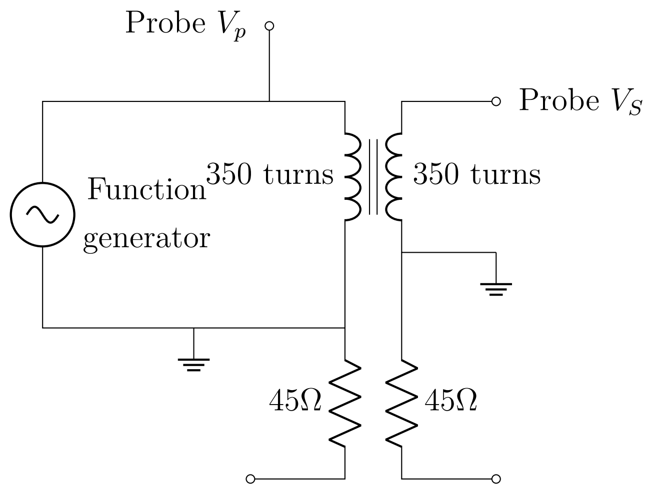
It would appear that adding a conductive core increases the mutual inductance, as if the iron is better able to conduct magnetic flux than air.

The iron cores introduce a significant dependence of M on the frequency. This is most likely due to Eddy currents induced in the core itself, taking away from the flux that can be used to induce the desired current in the other coil. Since the iron is conductive electrically, the Eddy currents are more prominent than with air alone.

Furthermore, the wires appear to increase M more than the solid bar. Eddy currents could also help to explain this. The solid bar allows for broad loops of Eddy current to appear, whereas the bundle of wire restricts the unwanted currents to small loops, leaving more magnetic flux available to induce the desired current in the other core.

Finally, the transformer is known to not be perfectly ideal, but its “closeness” to ideality can be determined using another circuit.

Figure 9: Circuit schematic for transformer



This circuit directly measures V_p, in addition to V_S as before. The relation between V_p and V_S was observed for all three core types. The more ideal the transformer, the closer $\frac{V_S}{V_p}$ should be closer to 1.

Table 3: Ideality observations for different cores

Core	$V_p/\mathbf{V}$	$V_S/\mathbf{V}$	V_S/V_p
Air	2.40 **V**	0.488 **V**	0.203
Iron Bar	6.56 **V**	3.68 **V**	0.561
Wire Bundle	8.24 **V**	5.60 **V**	0.680

It can be seen that the iron bar and wire bundle both make the transformer act more ideally, with the wire bundle being most ideal. This makes sense and agrees with the face that the bar and bundle increase the mutual inductance of the transformer.